

A rapid method for counter-flow heat regenerator calculation

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Abstract

A new rapid method of counter-flow heat regenerators performance prediction is presented. This method is based on analytical equation (15) which can be applied for regenerators operating in broad range of period time length. The accuracy of the method has been compared for slim regenerators with the robust method by Hill and Willmott [A. Hill, A.J. Willmott, A robust method for regenerative heat exchanger calculations, *Int. J. Heat Mass Transfer* 30 (1987) 241] and for corpulent regenerators with the solution by Hill and Willmott [A. Hill, A.J. Willmott, Accurate and rapid thermal regenerator calculations, *Int. J. Heat Mass Transfer* 32 (1989) 465].

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1. Introduction

Heat regenerators are periodic devices organizing heat transfer from hot to cold fluids. A solid matrix is heated by the hot fluid flowing along the matrix during half of the regenerator period, and subsequently during the second periodic half a cold fluid flowing along this matrix is heated. The geometry and properties of the matrix, as well as the fluids properties and the time length of the period, determine the regenerator performance. The theory of heat regenerators has a long history. First mathematical solutions published in the 1920s by Heiligenstedt [1,2], Nusselt [3,4], Hausen [5], Rummel [6], Schack [7] and Anzelius [8] initiated the search for analytical methods of regenerators calculation. Later publications by Gdula [9], Bes [10] and Tomczek [11–15] can be also included into this group. The progress of the computer technique enabled application of numerical methods to regenerators modelling, where worth mentioning are the three research groups: Willmott [16,17], Szargut [18–20] and BISRA [21]. How-

ever, because of the long computational time the search for improved mathematical methods continued [22–25]. A special problem in the regenerator modelling is the heat conduction within the matrix in the direction of fluids flow which for rotary regenerator has been initiated by Hahne-mann [26].

Three dimensional unsteady temperature field in the regenerators, and consequently long computing time are the reason that analytical solutions are still attractive for regenerators design. The price we pay for the elegance of the analytical solutions is the simplification of the process. There are two distinct problems in this approach: sophistication of the mathematical methods and the accuracy of the process description. Very often the experimental validation of the models is neglected.

The aim of the paper is to present a new rapid method enabling accurate prediction of the heat regenerator performance in pseudo-steady state of operation. The method is applicable in practical range of modern regenerator parameters. The accuracy of the method has been compared with the robust method by Hill and Willmott [24,25] and the differences are very small. The regenerator model considered is linear but the simplicity of equations makes them very suitable for designing.

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Nomenclature

<i>A</i>	heat transfer surface area, m ²	κ	dimensionless heat accumulated in the matrix
<i>B</i>	slimness of regenerator	λ	thermal conductivity, W m ⁻¹ K ⁻¹
<i>Bi</i>	Biot number	Θ	dimensionless temperature
<i>c_p</i>	specific heat capacity, J kg ⁻¹ K ⁻¹	ρ	density, kg m ⁻³
<i>Fo</i>	Fourier number		
<i>H</i>	regenerator length (height), m	<i>Subscripts</i>	
<i>i</i>	number of periods from start-up	c	cold half period
<i>m</i>	fluid mass flow, kg s ⁻¹	f	fluid
<i>R</i>	dimensionless coordinate across the matrix	h	hot half period
<i>r</i>	coordinate across the matrix, m	l	longitudinal
<i>T</i>	temperature, K	m	medium value
<i>t</i>	time, s	p	period
<i>w</i>	velocity, m s ⁻¹	s	solid
<i>Z</i>	dimensionless coordinate along the matrix	o	short regenerator
<i>z</i>	coordinate along the matrix, m	0	fluid inlet
<i>Greek symbols</i>			
α	convective heat transfer coefficient, W m ⁻² K ⁻¹		
δ	matrix plate half thickness (or half diameter), m		

2. Model equations

The real shapes of the solid matrix of an regenerator can be treated as formed by a number of identical matrix elements of regular shapes: plates, cylinders, balls. The temperature of the solid matrix element is assumed then to be a function of only two geometrical coordinates and the time $T(r, z, t)$, while the fluids temperature is described by a function of only one geometrical coordinate along which the flow takes place, and the time $T_f(z, t)$. The matrix temperature can be obtained by solution of the Fourier–Kirchhoff equation

$$\rho_s c_{ps} \frac{\partial T_s(r, z, t)}{\partial t} = \vec{\nabla} [\lambda_s \vec{\nabla} T_s(r, z, t)] \tag{1}$$

and the energy equation for the fluids has a form

$$m_f c_{pf} \frac{\partial T_f(z, t)}{\partial z} = \frac{A}{H} \alpha [T_s(\delta, z, t) - T_f(z, t)]. \tag{2}$$

The main boundary equations are: the solid matrix

$$\left. \frac{\partial T_s}{\partial t} \right|_{r=0} = 0, \quad -\lambda_s \left. \frac{\partial T_s}{\partial z} \right|_{r=\delta} = \alpha [T_s(\delta, z, t) - T_f(z, t)], \tag{3}$$

the fluids for co-flow

$$\begin{aligned} T_f(0, t) &= T_{h0} \quad \text{for } it_p \leq t < it_p + t_h \quad \text{and} \\ T_f(0, t) &= T_{c0} \quad \text{for } it_p + t_h \leq t < (i + 1)t_p, \end{aligned} \tag{4}$$

the fluids for counter-flow

$$\begin{aligned} T_f(0, t) &= T_{h0} \quad \text{for } it_p \leq t < it_p + t_h \quad \text{and} \\ T_f(H, t) &= T_{c0} \quad \text{for } it_p + t_h \leq t < (i + 1)t_p. \end{aligned} \tag{5}$$

The initial condition has a form: $T_s(r, z, 0) = T_f(z, 0) = T_{c0}$.

The periodic changes of the inlet fluids temperature (Eq. (4) or (5)) are the reason that regenerators are not able to reach a steady state. After sufficient number of periods a pseudo-steady state can be identified in which the temperature field within the solid matrix and fluids is identical within each period.

There are two very important problems analyzed in literature: • the role of reversal time during which both fluids are present within the solid matrix, and • the role of longitudinal heat conduction within the solid matrix. The reversal time most often is considered by introduction of an apparent time which is a difference of the real time and the value of the ratio H/w_{fm} [27]. The heat conduction along the matrix is usually neglected, however, it will be shown bellow that this is not justified in modern regenerators in which the matrix is made of materials characterized by high thermal conductivity. The argument that the fluids contact surface area with the matrix fronts (inlet and outlet) is smaller than 1% of the total heat transfer surface area does not justify negligence of the longitudinal heat conduction. In case of high values of λ_s the matrix temperature distribution is strongly influenced by heat conduction along the matrix regardless of the heat transfer conditions at the front surfaces, which is true even if adiabatic conditions are assumed at these surfaces.

3. Analytical solution for pseudo-steady state

Assuming constant properties of the solid matrix and the fluids, it is possible to write all the above equations in a dimensionless form

$$\frac{\partial \Theta_s}{\partial Fo} = \nabla^2 \Theta_s, \quad (6)$$

$$\frac{\partial \Theta_f}{\partial Z} = B[\Theta_s(1, Z, Fo) - \Theta_f(Z, Fo)]. \quad (7)$$

The boundary conditions for counter-flow have a form: the solid matrix

$$\left. \frac{\partial \Theta_s}{\partial R} \right|_{R=0} = 0, \quad (8)$$

$$\left. \frac{\partial \Theta_s}{\partial R} \right|_{R=0} + Bi[\Theta_s(1, Z, Fo) - \Theta_f(Z, Fo)] = 0, \quad (9)$$

the fluids

$$\begin{aligned} \Theta_f(0, Fo) &= 1 \quad \text{for } it_p \leq t < it_p + t_h \quad \text{and} \\ \Theta_f(1, Fo) &= 0 \quad \text{for } it_p + t_h \leq t < (i+1)t_p. \end{aligned} \quad (10)$$

Initial conditions can be written in a form

$$\Theta_s(R, Z, 0) = \Theta_f(Z, 0) = 0. \quad (11)$$

The dimensionless parameters in Eqs. (6)–(10) are

$$\begin{aligned} R &= \frac{r}{\delta}, \quad Z = \frac{z}{H}, \quad Fo = \frac{\lambda_s t}{\rho_s c_{ps} \delta^2}, \quad B = \frac{A\alpha}{m_f c_{pf}}, \\ Bi &= \frac{\alpha \delta}{\lambda_f}, \quad \Theta_s = \frac{T_s - T_{c0}}{T_{h0} - T_{c0}}, \quad \Theta_f = \frac{T_f - T_{c0}}{T_{h0} - T_{c0}}. \end{aligned}$$

The performance of a regenerator is well described by the temperature of the cold fluid at the outlet. Because this temperature varies with time, then a medium value is used by the designers that for regenerators, in which longitudinal heat conduction is neglected, can be described by equation given in [13]

$$\Theta_{fc,m} = \kappa(Fo_p, Bi, B) \frac{B_c}{Fo_c Bi_c}, \quad (12)$$

where the function $\kappa(Fo_p, Bi, B)$ means a dimensionless heat accumulated by the solid matrix within one period. The value κ is defined as a ratio of the heat accumulated during the heating half period to the maximum value attainable if the matrix was heated from the inlet temperature T_{c0} of the cold fluid to the inlet temperature of the hot fluid. The longer the regenerator period time the closer to unity is the $\kappa(Fo_p, Bi, B)$ value, that however does not mean that long periods favour optimal regenerator performance.

Most important is the cold fluid preheating $\Theta_{fc,m}$ which in contrary increases for shorter periods. The dimensionless heat accumulated κ is then a function of three parameters: Fo , Bi and B . For a corpulent regenerator $B \approx 0$ this function was found already by Hausen [27], Gdula [9] and Tomczek [11] for a plate type matrix. In case of a symmetrical regenerator ($Fo_h = Fo_c = 1/2 Fo_p$ and $Bi_h = Bi_c = Bi$) a simple expression can be obtained [11]

$$\kappa_0(Fo_p, Bi) = \sum_{k=1}^{\infty} A_k(\mu_k) \operatorname{tgh}\left(\frac{Fo_p \mu_k^2}{4}\right), \quad (13)$$

where the coefficients A_k for three regular matrix geometries are given in Table 1.

A simple analytical expression describing the dimensionless heat accumulated can be obtained only if the temperature variation across the matrix can be neglected. This is particularly justified for matrixes applied in modern regenerators. In such case for $B = 0$ and a symmetrical regenerator the solution of the periodic temperature field can be found, which enables to calculate the heat accumulated in a dimensionless form

$$\kappa_0(F_p) = \frac{(1 - \exp(-F_p/2))^2}{1 - \exp(-F_p)}, \quad (14)$$

where $F_p = Fo_p \cdot Bi = \alpha \cdot t_p / (\rho_s \cdot c_{ps} \cdot \delta)$ and $t_p = t_h + t_c$. For very short periods $F_p \rightarrow 0$ the value of $\kappa_0 \rightarrow 0$, while for long periods $F_p \rightarrow \infty$ the dimensionless heat accumulated within the matrix tends to unity. Eq. (14) was first proposed by Heiligenstaedt [28] who for large Bi values developed also a correction factor. This, however, is not necessary if the function κ_0 is calculated on basis of Eq. (13) which produces proper results for any Fo_p and Bi parameters.

For slimmer regenerators having $B > 0$ a solution of $\kappa(Fo_p, Bi, B)$ was found by Tomczek for co-flow [12] and for counter-flow [13]. In case where the temperature variation across the matrix can be neglected and $B > 0$ a robust method for κ calculation has been presented by Hill and Willmott [24,25] in which, however, no analytical equation is given. This topic was tackled earlier by Tomczek [14] who was able to calculate for a similar regenerator the performance with good accuracy.

Table 1
Coefficients A_k in Eq. (13) for solid matrix of three geometries

Matrix geometry	A_k	Eigenvalues	Source
Ball	$6 \frac{\mu_k^2 + (1 - Bi)^2}{\mu_k^2 + (1 - Bi)^2 - (1 - Bi)} \left(\frac{\sin \mu_k}{\mu_k^2} - \frac{\cos \mu_k}{\mu_k} \right)^2$	$\mu_k \operatorname{ctg} \mu_k = 1 - Bi$	[15]
Plate	$2 \frac{\mu_k^2 + Bi^2}{\mu_k^2 + Bi^2 + Bi} \left(\frac{\sin \mu_k}{\mu_k} \right)^2$	$\operatorname{ctg} \mu_k = \frac{\mu_k}{Bi}$	[9,11]
Cylinder	$4 \frac{Bi^2}{Bi^2 + \mu_k^2} \frac{1}{\mu_k^2}$	$\frac{J_0(\mu_k)}{J_1(\mu_k)} = \frac{\mu_k}{Bi}$	[15]

The mean preheat temperature determined by Eq. (12) is a function of three parameters: Fo , Bi and B . In case of symmetrical regenerators this function calculated by Tomczek [13] for plate type matrix on basis of a formal solution for κ is presented in Fig. 1 on example of two values of $B = 1$ and 5. It can be seen that as the period time becomes smaller the preheat $\Theta_{fc,m}$ (at constant B and Bi) tends to an asymptotic value (dashed line) determined by the function $B/(2 + B)$, known also in literature [24,25]. The four solid lines for each B , representing different values of Biot number, enable to examine the influence of designing parameters on regenerator performance. For example, let us consider a regenerator having: $B = 1$, $Fo_p/2 = 1$ and $Bi = 2$. Decreasing the matrix plate thickness by a factor 2 we increase the Fourier number four times and at the same time the Biot number becomes twofold smaller. Thus in consequence the mean preheat temperature $\Theta_{fc,m}$ changes slightly from 0.223 to 0.191. Increasing the regen-

erator slimness from $B = 1$ to 5 at constant $Fo_p/2 = 1$ and $Bi = 2$ we get higher preheat equal 0.599 instead of 0.223.

Analysing the curves presented in Fig. 1 we can notice that in a symmetrical regenerator the dimensionless cold fluid mean preheat temperature $\Theta_{fc,m}$ can be described by a single equation

$$\Theta_{fc,m} = 2 \frac{B}{F_p} \kappa(F_p, B) = 4 \frac{B}{2 + B} \frac{\kappa_0(F_p/B)}{F_p/B}, \quad (15)$$

where $F_p = \alpha \cdot t_p / (\rho_s \cdot c_{ps} \cdot \delta)$ and the function $\kappa_0(F_p/B)$ is defined by Eq. (14) in which the argument F_p should be replaced by (F_p/B) . Eq. (15) together with Eq. (14) can be applied also for regenerators with matrixes having thick walls but the accuracy becomes smaller as the Biot number increases. In such cases we can assume $F_p = Fo_p \cdot Bi$, however, the results are burden with error depending on Biot number. Applying Eq. (13) for $\kappa_0(F_p/B)$ in Eq. (15) we get accurate $\Theta_{fc,m}$ values also for large Bi numbers. The function $\kappa_0(F_p/B)/(F_p/B)$ tends to the value of 1/4 for very short cycle time $F_p \rightarrow 0$, so the limit mean cold fluid preheat can be calculated from a simple relation $\Theta_{fc,m} = B/(2 + B)$.

4. Discussion

The accuracy of the proposed rapid method has been examined first on basis of the cold fluid exit temperature $\Theta_{fc,m}$ from slim regenerators by comparison with the Hill and Willmott [24] solution, and the results are given in Table 2. The analyzed in Table 2 range of slimness ($B_h = B_c = 100-500$) is somewhat unrealistic, nevertheless it has been considered because Hill and Willmott [24] claimed, that their robust method can deal with such cases in contrary to other methods. As can be seen in Table 2 the difference between the two methods is very small even for the largest value of B . The proposed rapid method does not show any influence of the considered cycle time F_p on the preheat values, while the Hill and Willmott results [24] demonstrate small variation of $\Theta_{fc,m}$. This probably is the consequence of the numerical procedure applied in [24], because the values of $\Theta_{fc,m}$ for $(F_p/2) = 0$ are slightly over predicted in [24].

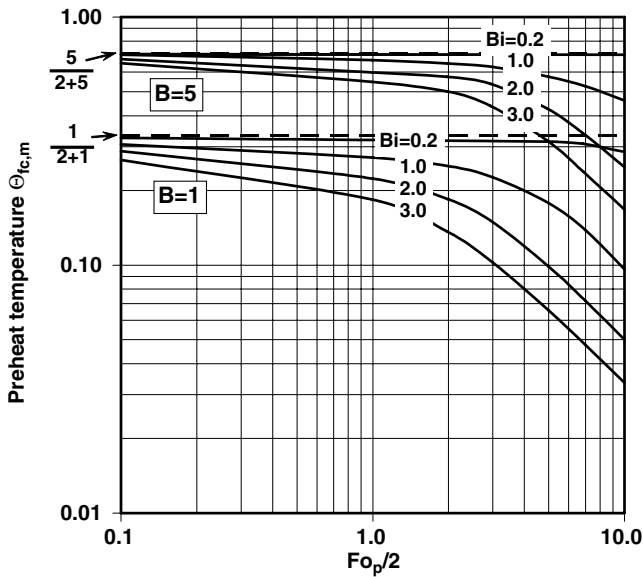


Fig. 1. Mean preheat temperature for symmetrical counter-flow regenerator as function of: $Fo_h = Fo_c = Fo_p/2$, $Bi_h = Bi_c = Bi$, $B_h = B_c = B$ [13].

Table 2
Comparison of calculated cold fluid preheat in a counter-flow slim regenerator

$F_h = F_c = F_p/2$	$\Theta_{fc,m}$					
	$B_h = B_c = B = 100$		$B_h = B_c = B = 250$		$B_h = B_c = B = 500$	
	Present method Eqs. (14) and (15)	Hill and Willmott [24]	Present method Eqs. (14) and (15)	Hill and Willmott [24]	Present method Eqs. (14) and (15)	Hill and Willmott [24]
0.000	0.98039	0.98088*	0.99206	0.99226*	0.99602	0.99611*
0.025	0.98039	0.98064	0.99206	0.99216	0.99602	0.99606
0.050	0.98039	0.98039	0.99206	0.99206	0.99602	0.99601
0.075	0.98039	0.98015	0.99206	0.99196	0.99602	0.99596
0.100	0.98039	0.97990	0.99206	0.99186	0.99602	0.99591
* should be:		0.98039		0.99206		0.99602

Table 3
Comparison of calculated cold fluid preheat in a counter-flow corpuent regenerator

$F_h = F_c = F_p/2$	$\Theta_{fc,m}$					
	$B_h = B_c = B = 1.0$		$B_h = B_c = B = 5.0$		$B_h = B_c = B = 10.0$	
	Present method Eqs. (14) and (15)	Hill and Willmott [25]	Present method Eqs. (14) and (15)	Hill and Willmott [25]	Present method Eqs. (14) and (15)	Hill and Willmott [25]
0.5	0.32656	0.3304	0.71369	0.7134	0.83316	–
1.0	0.30808	0.3221	0.71191	0.7109	0.83264	0.8322

Table 3 presents comparison of the mean preheat temperature calculated by the proposed method with the Hill and Willmott [25] results for corpuent regenerators. It can be seen that for the very small $B \approx 1$ value utilization of Eq. (15) or (16) leads to a largest difference between the methods. The $B \approx 1$ value is, however, of no practical use. In case of larger more practical $B > 5$ the two methods produce very good agreement.

The relation between the $\Theta_{fc,m}$ and B is presented in Fig. 2 for regenerators in which the temperature variation across the solid matrix is neglected. Three lines for different period time F_p are presented, including the line representing the limit case for $F_p \rightarrow 0$. A sharp increase of the cold fluid preheat can be observed for small $B \leq 20$, above which the length of the regenerator operating with short time periods has only small influence on the preheat. For longer periods $F_p = 40$ we observe considerable preheat increase until slinness $B \approx (40-50)$. The role of the period time on preheat is clearly seen in Fig. 3. Shortening the cycle time can effectively influence the preheat only to the value $F_p/B \approx 1$ below which almost no preheat can be gained. For the slinness $B = 5$ it is possible to compare the $\Theta_{fc,m}$ values calculated by Eqs. (15) and (14) with that given in Fig. 1. The new rapid methods slightly over predicts the results in Fig. 1 for large Biot numbers $Bi > 1$

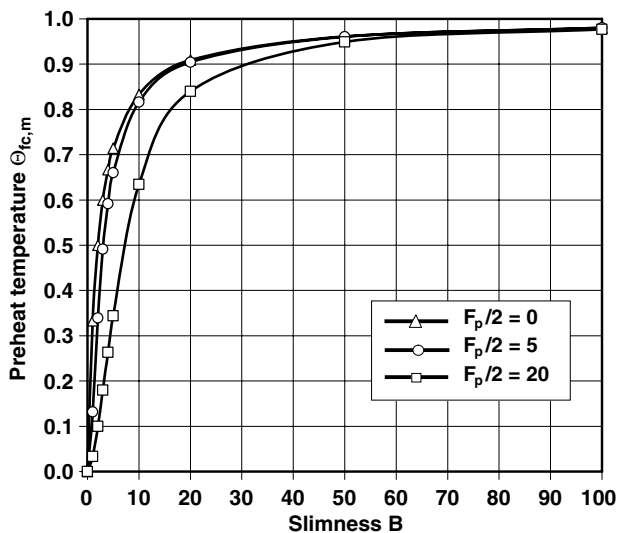


Fig. 2. Mean preheat temperature calculated by Eqs. (14) and (15) for symmetrical counter-flow heat regenerator as function slinness and period time.

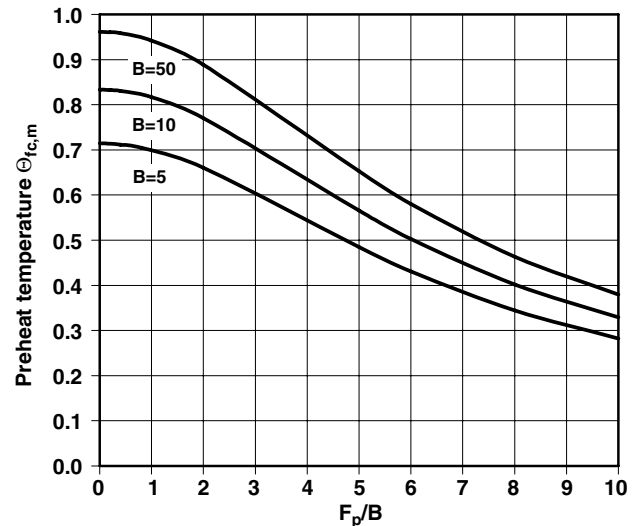


Fig. 3. Mean preheat calculated by Eqs. (14) and (15) for symmetrical counter-flow heat regenerator as function of period time for three values of slinness.

but for smaller Bi the accuracy is very good. Applying Eq. (13) instead of (14) in Eq. (15) we get exactly the same results as presented in Fig. 1.

The asymmetry of regenerators operation ($Fo_h \neq Fo_c$, $Bi_h \neq Bi_c$ and $B_h \neq B_c$) in practical cases is not large. Usually, because the solid matrix material thermal properties as a function of temperature are not known exactly, then we can assume $Fo_h = Fo_c$. For the two other parameters a ratio can be observed: $Bi_h/Bi_c = (1.1-1.5)$ and $B_h/B_c = (0.9-1.3)$. In this range of value it is possible to treat the regenerator as a symmetrical device with the mean parameters, calculated for example according to Hausen [27,25] method.

The role of the heat conduction within the solid matrix along the direction of fluids flow (z coordinate) on the regenerator performance has been examined numerically [29]. Two regenerators built of the same material (SiSiC), having coefficient of heat conduction $\lambda_s = 150$ W/(m K), were considered: (a) $Fo = 44$, $B = 4.7$, $Bi = 3 \times 10^{-3}$; (b) $Fo = 427$, $B = 12$, $Bi = 1.3 \times 10^{-3}$. The hot fluid inlet temperature was assumed $T_{h0} = 1200$ °C equal in both cases. The regenerators operated in a counter-flow mode. It has been found that taking into account the heat conduction along the fluids flow direction abates the preheat temperature $\Theta_{fc,m}$ by about 3.8% in case (a) and by 11.6% in case

(b), that should be remembered when the accuracy of regenerator modelling is considered. The computed reduction of the preheat $\Theta_{fc,m}$ can be compared qualitatively with the values from the graphs published by Bahnke and Howard [30] presenting the effect of heat conduction within the matrix along the fluid flow direction in rotary regenerators for the conduction parameter $K = (\lambda_{sl}A_{sl}) / (C_f H)$ in which A_{sl} means the solid matrix cross-section area conducting heat in longitudinal direction and $C_f = m_f c_{pf}$. London [30,31] proposed for $K < 0.1$ and $B > 10$ a simplified relation enabling correction of the cold fluid preheat for longitudinal conduction

$$\frac{\Theta_{fc,m}(\lambda_{sl} \neq 0)}{\Theta_{fc,m}(\lambda_{sl} = 0)} \cong 1 - K. \quad (16)$$

For the considered in [29] two regenerators the conduction parameter was equal: $K_a \approx 0.05$ for regenerator (a) and $K_b \approx 0.1$ for regenerator (b). Thus the simple relation (16) is a good engineering approximation. The same parameter K was also used for testing of the longitudinal heat conduction during single fluid blow in the regenerator [32].

5. Conclusions

The presented method based on analytical Eq. (15) allows for rapid calculation of the pseudo-steady state of counter-flow symmetrical heat regenerators in broad range of parameters.

The strength of the method is the speed of regenerator performance calculation, while the weakness comes from the model linearity. This, however, may be partially overcome by applying iterative procedure allowing for some nonlinearity caused by the temperature dependence of thermal properties.

Comparison of the mean air preheat temperature calculated by the proposed method with those published earlier [24,25] shows satisfactory accuracy for both corpulent and slim regenerators.

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